Admissible Determinacy

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June 14, 2013

Games

We define the Gale-Stewart game played over ${}^{\omega}\omega$ with payoff set $A \subset {}^{\omega}\omega$, denoted $G(A; {}^{<\omega}\omega)$ as follows:

We have two players, *I* and *II*, and an inning for each natural number. At inning 0, player *I* begins by picking a natural, and player *II* responds in inning 1 by picking a natural number. From there, the players alternate picking natural numbers. It is important to note that each player has perfect information, and can see all moves made prior.

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We end up with the following, where $x_i \in \omega$

$$\begin{array}{llllllll} I: x_0 & & x_2 & & \dots & & x_{2n} & & \dots \\ II: & x_1 & & x_3 & & \dots & & x_{2n+1} \end{array}$$

We say that $\bar{x} = \langle x_0, x_1, \dots \rangle$ is a play of the game. I wins if $\bar{x} \in A$, and II wins otherwise.

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Games

A strategy for *I* (resp. *II*) in $G(A; {}^{<\omega}\omega)$ is a function σ from positions where it is *I*'s turn (resp. *II*'s turn) to ω. One way of thinking of a strategy is as a function that tells *I* (resp. *II*) what to play when it is her turn. A Strategy σ is said to be winning for *I* (resp. *II*) if every play consistent with σ is a win for *I* (resp. *II*). A set $A \subset {}^{\omega}\omega$ is said to be determined if either player has a winning strategy for $G(A; {}^{<\omega}\omega)$. In particular, we are interested in the determinacy of all sets in a given pointclass.

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Determinacy Hypotheses

The *Axiom of Determinacy*, abreviated AD and introduced by Mycielski and Steinhaus in 1962, states that all such games $G(A; {}^{<\omega}\omega)$ are determined. Note that a simple diagonal argument will show that AD is inconsistent with the statement "$\mathbb{R}$ is well-orderable". Since most mathematicians enjoy the free use of Choice, AD is not a very good candidate for a new axiom. With this in mind, it is natural to turn to "reductions" of AD.

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Determinacy Hypotheses

The following are some reductions of AD that have been considered:

- Determinacy of all games $G(A; {}^{<\omega}\omega)$ where A is a Π^1_1 subset of ${}^{\omega}\omega$.
- Determinacy of all games $G(A; {}^{<\omega}\omega)$ where A is a Projective Subset of ${}^{\omega}\omega$.
- Determinacy of all games $G(A; {}^{<\omega}\omega)$ where A is an ordinal definable subset of ${}^{\omega}\omega$.
- AD holding in $L(\mathbb{R})$.

Each of the above determinacy hypotheses can be derived from ZFC+“there exists a large cardinal LC” for varying LC.

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Admissible Sets

This talk is about a particular reduction of AD, Admissible Determinacy. Before going further, we will review the definition of Kripke Platek set theory.

Definition

Kripke-Platek is the theory with the following axioms:

1. Extensionality
2. Pairing
3. Union
4. Foundation in the form of $\in$-induction
5. Δ_0-Comprehension: Δ_0-definable subclasses of sets are sets.
6. Δ_0-Collection: Point-wise images of sets by ordinal-valued Δ_0 functions are bounded.

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Admissible Sets

A transitive class M is said to be *admissible* if $(M; \in) \models KP$. It is a-admissible if $(M; \in; a \cap M) \models KP$. Note that admissible sets have stronger properties than ones stated in the definition.

Lemma

Let $(M; \in)$ be admissible, then $M \models \Delta_1$-comprehension and Σ_1-collection.

An ordinal α is an x-admissible ordinal if $L_\alpha[x]$ is admissible. We use $\omega_1^{\mathrm{CK}}(x)$ to denote the least x-admissible ordinal. In order to avoid clutter, we will use M_x to denote $L_{\omega_1^{\mathrm{CK}}(x)}[x]$.

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Large Cardinal Companions

It turns out that one can measure the consistency strength of "natural" determinacy hypotheses in terms of the large cardinal hierarchy.

Definition

Let ϕ be a formula in the language of set theory. We say that a large cardinal axiom, denoted LC_ϕ, is the *large cardinal companion* of ϕ if ZFC+LC_ϕ and ZFC+ϕ are equiconsistent.

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Admissible Determinacy

We now turn to the subject of this talk, a particular reduction of AD known as Admissible Determinacy.

Definition

Admissible Determinacy is the hypothesis that there is an $x \in {}^{\omega}\omega$ such that M_x satisfies the statement "Every ordinal definable class of reals is determined".

Note that Admissible Determinacy ranges over classes of reals. This is because we are working inside a model that does not satisfy comprehension, so not every ordinal definable class of reals is a set. Our interest here is in the Large Cardinal Companion of Admissible Determinacy. This was first considered by Andy Lewis.

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Admissible Determinacy

Finding the large cardinal companion of a particular determinacy hypothesis turns out to be a bit tricky. One way to approach this problem is by breaking it into two parts, that of finding a good upper bound and a good lower bound. The hope is that these upper and lower bounds will match, but this is not the case for known upper and lower bounds of Admissible Determinacy.

Open Problem

What is the Large Cardinal Companion of Admissible Determinacy?

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Approaching the Problem: the Upper Bound

One approach is to prove the relative consistency of Admissible Determinacy level-by level. We begin by noting that the assertion that $M_x \models$ OD-Determinacy can be reformulated as the statement that, $(\forall n < \omega)(M_x \models \mathrm{OD}_{\Sigma_n}$-Determinacy). This is the statement that, for each n, every class of reals that is definable from a Σ_n formula with ordinal parameters in M_x, is determined in M_x.

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Approaching the Problem: the Upper Bound

Our goal is now to find, for each n, LC_n that proves the relative consistency of ”There is a real x such that $M_x \models OD_{\Sigma_n}$-Determinacy”. However, we need to be careful to make sure that we can amalgamate the LC_n (and the reals) somehow. One way to get such LC_n is to ask that they each prove the existence of a κ that admits some very strong measures. In addition, in order to ensure that the reals can be amalgamated as well, we can ask that LC_n proves the above for a Turing cone of reals. This is the approach that Diego Rojas and Andrés Caicedo used to find the best known upper bound.

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The Mitchell Order

Recall:

Definition

For a cardinal κ if there is a κ-complete, non-principal, normal ultrafilter $\mathcal{U}$ on κ, we say κ is measurable, and we call $\mathcal{U}$ a measure on κ.

If $\mathcal{U}$ is a measure on κ, the ultrapower by $\mathcal{U}$ induces an elementary embedding $i_{\mathcal{U}} : V \xrightarrow{\prec} Ult(V;\mathcal{U})$ with critical point κ and $Ult(V;\mathcal{U})$ transitive.

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Given a measure $\mathcal{U}$ on a cardinal κ, we see that $\mathcal{U} \notin Ult(V; \mathcal{U})$. However, suppose we have two measures $\mathcal{U}$ and $\mathcal{V}$ on κ with the additional property that $\mathcal{U} \in Ult(V; \mathcal{V})$. Then we have:

1. $i_{\mathcal{U}}(\kappa) < i_{\mathcal{V}}(\kappa)$;
2. $V \models \kappa$ is the limit of measurables, in fact;
3. $V \models \{\alpha < \kappa : \alpha \text{ is measurable}\}$ is stationary in κ.

We see that in some sense, $\mathcal{V}$ is a stronger measure than $\mathcal{U}$. In particular, 1) allows us to put a well-founded partial order on measures on κ given by

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1. $i_{\mathcal{U}}(\kappa) < i_{\mathcal{V}}(\kappa)$;
2. $V \models \kappa$ is the limit of measurables, in fact;
3. $V \models \{\alpha < \kappa : \alpha \text{ is measurable}\}$ is stationary in κ.

We see that in some sense, $\mathcal{V}$ is a stronger measure than $\mathcal{U}$. In particular, 1) allows us to put a well-founded partial order on measures on κ given by

$$\mathcal{U} \lhd \mathcal{V} \Leftrightarrow \mathcal{U} \in \mathit{Ult}(V; \mathcal{V})$$

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We now define a rank function using $\triangleleft$ on measurable cardinals κ, and their witnessing measures.

Definition

Let κ be a measurable cardinal, and let $\mathcal{U}$ be a measure on κ. The Mitchell Order of $\mathcal{U}$, denoted $o(\mathcal{U})$ is

$$o(\mathcal{U}) := \sup\{o(\mathcal{V}) + 1 : \mathcal{V} \triangleleft \mathcal{U}\}$$

The Mitchell Order of κ is:

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The Mitchell Order gives us a hierarchy of measurables to work with, with κ being measurable if and only if $o(\kappa) \geq 1$ forming the base of this hierarchy. This hierarchy does have a limit though, because for any measure $\mathcal{U}$ on κ, $\mathcal{U} \in \mathcal{P}^2(\kappa)$. This gives us that $|\{\mathcal{U} : \mathcal{U} \text{ is a measure on } \kappa\}| \leq 2^{2^\kappa}$, which in turn tells us that $o(\kappa) \leq 2^{2^\kappa}$. In particular we see that, under the GCH, $o(\kappa) \leq \kappa^{++}$. The upper bound on Admissible Determinacy is in the neighborhood of the existence of cardinals κ such that $o(\kappa) = \kappa^{++}$.

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The Upper Bound

Theorem (Caicedo, Rojas)

Assume that $(\forall n < \omega)$ *there is a model of the form* $L_\alpha[\vec{\mathcal{U}}]$*, such that:*

1. $L_\alpha[\vec{\mathcal{U}}] \models \mathrm{ZF}^- + \vec{\mathcal{U}}$ *is a coherent sequence of measures;*
2. $L_\alpha[\vec{\mathcal{U}}]$ *is iterable;*
3. $L_\alpha[\vec{\mathcal{U}}] \models (o(\kappa) = \kappa^{++} \wedge$ *the* $(n+1)$*st successor of* κ *exists)*

Then, for a cone of $x \in {}^\omega\omega$, $M_x \models \mathrm{OD}$*-Determinacy.*

The proof of this result uses ideas of Woodin, Martin, and Kechris on the relation between Woodin cardinals, Δ^1_2-determinacy, and OD-determinacy, and specializes them to the context of admissible sets via results of Neeman and Steel.

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Approaching the Problem: The Lower Bound

The upper bound for the "cone version" of Admissible Determinacy that we have is believed to be optimal. On the other hand, the lower bounds that we have are small by comparison. In order to find a lower bound, we begin by assuming Admissible Determinacy, and play a game that will "code" some Large Cardinal axiom. We need to make sure that this game never leaves our model M_x in addition to being ordinal definable. Here we see difficulties crop up, as we do not have as many tools as we would like when working in KP.

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Typically a lower bound LC for ϕ is established as follows:

1. We want to show that ϕ implies the existence of inner models that satisfy LC. We assume, towards a contradiction, that LC is not reached.
2. Using that LC is not reached, we show that the core model K exists, and satisfies certain properties. In particular, we want it to satisfy a version of covering.
3. We then show that the combinatorics of ϕ violate these properties. This gives us that inner models of LC exist.

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Unfortunately, these games (and the definability restrictions inherent to working in KP) do not seem strong enough to produce models with sequences of measures. Even $o(\kappa) = 2$ seems to be out of reach, whereas our upper bound is in the area of $o(\kappa) = \kappa^{++}$. That said, the games are successful enough to show the existence of models with many measurables.

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The Lower Bound

Before giving the lower bound, we need one last definition:

Definition

A cardinal κ is 0-*fixed* if κ is measurable. We say that κ is $(t+1)$-*fixed* if $\{\lambda < \kappa : \lambda \text{ is } t\text{-fixed}\}$ has cardinality κ.

We can now present the best known lower bound for Admissible Determinacy:

Theorem (Rojas)

Assume Admissible Determinacy, then for every $t \in \omega \setminus \{0\}$, there is a model of ZFC with a $(t-1)$-fixed cardinal.

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Closing Questions

We conclude with some questions in the area:

1. Can one use Martin games to produce a κ with $o(\kappa) = 2$?
2. Given Σ_n Admissible Determinacy, can we show that, for some $m < n$, there are many reals for which Σ_m determinacy holds?
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