STUDENT LOGIC COLLOQUIUM

Andrés Caicedo

will speak on

What are Super-real Fields?

on

Friday, October 6

in

9 Evans Hall

at

4:10 p.m.

Abstract

The study of rings of continuous functions has a long history, although the origin of their systematic treatment is usually identified with a paper of Hewitt in the TAMS (1948). The classical theory is covered in the 1960 monograph "Rings of Continuous Functions" by Gillman and Jerison (reprinted as GTM 43).

Let X be a topological space, and let $C(X)$ denote the ring of continuous real-valued functions with domain X. To study the algebraic properties of $C(X)$, a natural approach is to consider its ideals and the quotients induced by them; the prime ideals turn out to provide a particularly fruitful theory:

Suppose X is completely regular and P is a prime ideal in $C(X)$. Let

$$A_P = C(X)/P.$$

Then there is a natural total ordering of A_P, and A_P is a commutative unital integral domain. Let K_P be its quotient field.

A *super-real field* is an ordered field $K \not\cong \mathbb{R}$ which is isomorphic to some K_P. The class of super-real fields, introduced by Woodin and Dales in their 1996 book (LMS Monographs 14) generalizes that of hyper-real fields introduced by Gillman and Jerison and in particular the class of ultrapowers of $\mathbb{R}$.

In this talk I will mention some of their basic properties (for example, super-real fields are real-closed), some of their most distinguished sub-classes, and some quite intriguing open problems. For example:

Suppose X is compact and P is prime, non-maximal. Let K_P^{fin} denote the algebra of *finite* elements of K_P. For which ideals P is K_P^{fin} normable? When that is the case, there is a discontinuous homomorphism from $C(X,\mathbb{C})$ into a Banach algebra, but the existence of such a homomorphism is known to be independent of the standard axioms of set theory.

I will try hard to make this talk accessible to everyone.