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Review of
Constructing (ω_1, β) -morasses for $\omega_1 \leq \beta$
by Bernhard Irrgang
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In previous work, *Proposing (ω_1, β) -morasses for $\omega_1 \leq \beta$* (MR2827553), hereafter PM, Irrgang defined the notion of (ω_1, β) -morasses for $\omega_1 \leq \beta$. This extends Jensen’s definition (see *Higher-gap morasses*, Handwritten manuscript, 1972/73), that only considers the notion for $\beta < \omega_1$. The reasons for this limitation, and how to circumvent them, are explained in PM.

In PM, the notion of κ -standard morasses is also introduced. It is shown there that any $\omega_{1+\beta}$ -standard morass is an (ω_1, β) -morass, and that the existence of κ -standard morasses implies the existence of sets X such that $L_\kappa[X]$ computes cardinals correctly, and admits fine structure and condensation. See MR2827553 for details on these notions. Their definition is recalled in Section 1 of the paper under review.

The main result of this paper is that, conversely, if κ is a cardinal and $L_\kappa[X]$ satisfies these three properties, then there is a κ -standard morass (in $L_\kappa[X]$), see Section 3. In particular, the notion of (ω_1, β) -morasses is consistent for all $\beta \geq \omega_1$, and they exist in L .

Fine structure is reviewed in Section 2, and described in detail for $L[X]$ (applications, such as the existence of \square -sequences, are presented in Irrgang’s dissertation *Kondensation und Moräste*, München, 2002). The definitions of morasses and standard morasses are reviewed in Section 3. The construction is elaborate but resembles the definition of \square -sequences in L . The paper is carefully written and well-organized.